

Induced θ -Vacuum States in Heavy Ion Collisions: A Possible Signature.

Kirk Buckley , Todd Fugleberg and Ariel Zhitnitsky

Physics and Astronomy Department
University of British Columbia
6224 Agricultural Road, Vancouver, BC V6T 1Z1, Canada
e-mail: kbuckley@physics.ubc.ca
fugle@physics.ubc.ca
arz@physics.ubc.ca

Abstract:

It has been suggested recently [1] that an arbitrary induced θ -vacuum state (θ^{ind}) could be created in heavy ion collisions. If such a state can be created, it would decay by various mechanisms to the $\theta^{fund} = 0$ state which is the true ground state of our world. In the following we will discuss the possibility of studying this unusual state through the emission of pions, η -mesons, and η' -mesons. We will also present the spectrum of the produced particles in this non-zero θ background. We use the instantaneous perturbation theory for our estimates.

1 Introduction

The long awaited Relativistic Heavy Ion Collider at Brookhaven (RHIC) could yield many fascinating results concerning the accepted theory of the strong force, QCD. Although the main goal of this experiment is to study the quark-gluon plasma, there exist many other possibilities. One of the most sought after results is the nature of the chiral symmetry restoration and/or deconfining phase transition. Another possible outcome is the so called disoriented chiral condensate (DCC). In addition, it was recently suggested [1] that it may be possible to create an induced $\theta \neq 0$ -state in heavy ion colliders. If this is possible, the consequences could include the observation of a new state of matter.

Indeed, as is known, in the infinite volume limit and in thermal equilibrium the θ -vacuum state is the absolutely stable ground state of a new world with new physics quite different from ours. In particular, P and CP symmetries are strongly violated in this world. In spite of the fact that the θ -vacuum state has a higher energy (see below) this state is stable due to the super selection rule: There does not exist a gauge invariant observable \mathcal{A} in QCD, which would communicate between two different worlds, $\langle \theta' | \mathcal{A} | \theta \rangle \sim \delta(\theta - \theta')$. Therefore, there are no transitions between these worlds[2].

Of course, we do not expect that such a stable θ -state can be produced in heavy ion collisions. However, we do expect that locally, for a short period of time due to the non-equilibrium dynamics after the QCD phase transition, a large domain with wrong θ direction may be formed. This provides us with a unique opportunity to study a new state of matter. The idea is very similar to the old idea of the creation of the Disoriented Chiral Condensate (DCC) in heavy ion collisions [3] -[6]. DCC refers to regions of space (interior) in which the chiral condensate points in a different direction from that of the ground state (exterior), and separated from the latter by a hot shell of debris. Our starting point is the conjecture that a classically large domain with a nonzero $U(1)_A$ chiral phase may be formed in a heavy ion collisions, i.e. we assume that the expectation value for the chiral condensate in a sufficiently large domain has, in general, a non-zero $U(1)$ phase: $\langle \bar{\Psi}_L \Psi_R \rangle \sim e^{i\phi^{singlet}} |\langle \bar{\Psi} \Psi \rangle|$. This phase is identified with θ^{ind} . Such an identification is a direct consequence of the transformation properties of the fundamental QCD Lagrangian under $U(1)_A$ rotations when the chiral singlet phase can be rotated away at the cost of the introduction of the induced θ , $\theta^{ind} = -N_f \phi^{singlet}$.

We should remind the reader at this point that in QCD, the fundamental θ parameter enters in the combination $\theta Q = \theta \frac{\alpha_s}{8\pi} G^a \tilde{G}^a$. This gives us the first contribution to the physically observable parameter $\bar{\theta}$. The second contribution is related to the phase which is introduced upon diagonalization of the quark mass matrix, which can be in principle arbitrary. Current experimental results indicate that these two numbers cancel with precision better than 10^{-9} . The problem of why $\bar{\theta} \simeq 0$ in our world is known as the strong CP -problem, and we do not address this problem in the present paper. Our point is the following: by having a non-zero $U(1)$ phase $\langle \bar{\Psi}_L \Psi_R \rangle \sim e^{i\phi^{singlet}} |\langle \bar{\Psi} \Psi \rangle|$ in a sufficiently large region, we essentially break this precise cancellation between two contributions in a macroscopically large domain¹. Exactly this non-cancellation will mimic a non-zero θ -parameter.

¹We use the term macroscopic in this paper to refer to scales which are larger than the fundamental scale, Λ_{QCD}^{-1}

The production of non-trivial θ^{ind} -vacua would occur in much the same way as discussed previously for DCC[3]-[6]. The new element is that, in addition to chiral fields differing from their true vacuum values, the induced θ -parameter, which is zero in the real world, becomes effectively nonvanishing in the macroscopically large domain as explained above. Therefore, although the fundamental θ -parameter remains zero, we would expect that an induced θ -parameter can be non-zero in a macroscopically large domain, and it will mimic the physics of the world with $\theta^{fund} \neq 0$. A different name for the same object would be a “zero (spatially independent) mode of the η' field” which exactly corresponds to the flavor-singlet spatially independent part of the chiral condensate phase. However, we prefer to use the term θ^{ind} because the symbol η' is usually associated with the free asymptotic state of the heavy η' -meson (excitation) and not with a classical constant field (condensate) in a large domain where η' -mesons interact very strongly and change their properties in this background. This other terminology could be also misleading because of this reason. In addition to the above mentioned difference, the disoriented chiral condensate involves the amplification of the low momentum modes of the charged pions, while the formation the θ -state involves the amplification of the low momentum modes of only the neutral particles, including the η' singlet.

One might ask, “How could such a θ^{ind} -state be detected if it is created in heavy ion collisions for a very short period of time?”. One obvious possibility is that the θ^{ind} -state could be observed through Goldstone bosons with specific CP -odd correlations [7]. However, a more promising signature is related to the low momentum modes of the chiral fields[8]. Indeed, as was shown in [1], the creation of the θ^{ind} -state involves the enhancement of the low frequency modes of the neutral chiral fields. Therefore, it is natural to expect that particles made of these chiral fields are to be produced when the θ^{ind} -state blows apart. To be more precise, in the limit $m_u = m_d = m_s \ll \Lambda_{QCD}$ the only relevant physical degree of freedom is a singlet phase of the chiral condensate which transforms, at the very end of the transition exclusively, to the η' -mesons when the θ^{ind} -state falls apart. Therefore, in this limit we would expect an excess of η' -mesons to be produced. However, due to the mixing and difference in masses, $m_u \neq m_d \neq m_s$, it is expected that all neutral modes are to be produced. The main goal of the present work is to study the spectrum and the angular distribution of particles which will be produced when the θ^{ind} -state falls apart.

However, before we discuss some numerical estimates, we would like to present some estimates based on simple energetic considerations. As we discuss below (6), the vacuum energy density in the θ -state is greater than in the $\theta = 0$ vacuum state by the amount:

$$\Delta E = E_{vac}(\theta) - E_{vac}(\theta = 0) = 2m|\langle \bar{q}q \rangle|(1 - |\cos \frac{\theta}{2}|),$$

where for simplicity we use $N_f = 2$ with $m_u = m_d = m$. When the θ^{ind} -state blows apart, the energy associated with this background will be released as Goldstone bosons carrying the total energy:

$$\Delta E V \simeq 2m|\langle \bar{q}q \rangle|(1 - |\cos \frac{\theta}{2}|) \cdot V \simeq 20 \cdot \left(\frac{V}{(10 \text{ fm})^3} \right) \text{ GeV},$$

where $V = L^3$ is $3d$ volume of the θ^{ind} -background measured in *Fermi*. Therefore, given

the cm energy $\sqrt{s} = 40 \text{ TeV}$, only a small fraction:

$$\rho \sim \frac{\Delta EV}{\sqrt{s}} \sim \frac{20(\frac{V}{(10 \text{ fm})^3}) \text{ GeV}}{40 \text{ TeV}} \sim 10^{-3} \left(\frac{V}{(10 \text{ fm})^3} \right)$$

of the total collision energy will be released through the decay of the induced θ^{ind} -state. Two of the most profound features of the Goldstone bosons produced in this decay are the following:

1. Due to the fact that the θ^{ind} formation is caused by amplification of low momentum modes, we expect the spectrum of the Goldstone bosons from this decay to be strongly enhanced at low $|\vec{k}| \sim L^{-1}$;
2. Due to the nonzero value for the topological density $\langle \theta | \frac{\alpha_s}{8\pi} G\tilde{G} | \theta \rangle = -\frac{\partial V_{vac}(\theta)}{\partial \theta} = -m|\langle 0 | \bar{q}q | 0 \rangle| \sin \frac{\theta}{2}$, see (6), one could expect that some P , CP odd correlations would appear in this background.

Considering this, we should look for enhanced production of particles such as the π , η , and η' with low momentum, on the $(10 - 100) \text{ MeV}$ scale (depending on the size of the domain L). The difference between the θ -vacuum state and the DCC which also involves enhancement of low momentum modes is that the θ -vacuum state also produces η' -mesons. These should decay eventually by various processes to photons and dileptons, and if a large number of these low momentum particles are detected this would be a definite signal of the creation of a macroscopically large domain with $\theta^{ind} \neq 0$. One could further speculate that such an enhancement could also account for the unexplained large number of low momentum dileptons seen at CERN [9] as suggested in [8]. However, detailed calculations are needed to give this speculative conjecture further support.

The presentation is organized as follows. In Sect. 2 we briefly describe the effective Lagrangian constructed for QCD as given in [10, 11]. In Sect. 3 we demonstrate that it is theoretically possible to create $\theta^{ind} \neq 0$ -states in heavy ion collisions. The spectrum of the Goldstone bosons produced in the nonzero θ background is calculated in Sect. 4 for different geometries of the θ^{ind} -region. Finally, we end with concluding remarks and future considerations in Sect 5.

2 Low Energy Effective QCD Lagrangian

The starting point of our analysis is the low energy effective Lagrangian which reproduces the anomalous conformal and chiral Ward Identities. The corresponding construction in the large N_c limit has been known for a long time [12, 13]. The generalization of the construction of ref. [12, 13] for finite N_c was given in [10, 11], and we shall use formulae from the papers [10, 11]. However, we should remark at the very beginning that all local properties of the effective Lagrangians for finite and infinite N_c are very much the same. Small quantitative differences in local physics between the description of [12, 13] on the one hand and the description of [10, 11] on the other hand, do not alter the qualitative results which follow.

The effective Lagrangian describes the light matter fields of QCD which consists of an octet of pseudo-Goldstone bosons (π 's, K 's, and the η), and the η' singlet field. We

parameterize these fields by a unitary matrix U_{ij} corresponding to the γ_5 phases of the chiral condensate ($\langle \overline{\Psi}_R^i \Psi_L^j \rangle = -|\langle \overline{\Psi}_R \Psi_L \rangle| U_{ij}$) in the following way:

$$U = U_0 \exp \left[i\sqrt{2} \frac{\pi^a \lambda^a}{f_\pi} + i \frac{2}{\sqrt{N_f}} \frac{\eta'}{f_{\eta'}} \right], \quad (1)$$

where U_0 solves the minimization equation for the effective potential, π^a represents the pseudoscalar octet, and N_f is the number of flavors. The effective potential derived in [10, 11] takes the following form:

$$V(U, \theta) = -E \cos \left[\frac{1}{N_c} (\theta - i \log \text{Det} U) \right] - \frac{1}{2} \text{Tr} (MU + M^\dagger U^\dagger), \quad (2)$$

where M is the diagonal quark mass matrix defined with their condensates: $M = -\text{diag}(m_i \langle \overline{\Psi}_i \Psi_i \rangle)$ and $E = \langle b\alpha_s / (32\pi) G^2 \rangle$ is the vacuum energy with $b = 11N_c/3 - 2N_f/3$. Expanding the cosine (this corresponds to the expansion in $1/N_c$) we recover the result of [12] at lowest order in $1/N_c$ together with the constant term E required by the conformal anomaly:

$$V(U, \theta, N_c \rightarrow \infty) = -E - \frac{\langle \nu^2 \rangle_{YM}}{2} (\theta - i \log \text{Det} U)^2 - \frac{1}{2} \text{Tr} (MU + M^\dagger U^\dagger) + \dots \quad (3)$$

Here we used the fact that in the large N_c limit $E(\frac{1}{N_c})^2 = -\langle \nu^2 \rangle_{YM}$ where $\langle \nu^2 \rangle_{YM} < 0$ is the topological susceptibility in pure YM theory. Corrections in $1/N_c$ stemming from Eq.(3) constitute a new result of ref.[10]. In order to demonstrate that the effective potential shown above is of the correct form, we can verify that the following three features are characteristic of Eq.(2):

1. Eq.(2) correctly reproduces the Witten-Di Vecchia-Veneziano effective chiral Lagrangian [12] in the large N_c limit;
2. It reproduces the anomalous conformal and chiral Ward identities of QCD ;
3. It reproduces the known dependence in θ (*ie.* 2π periodicity of observables) [12].

To study the vacuum properties of the θ vacua it is convenient to choose a diagonal basis to parameterize the fields U as $U = \text{diag}(\exp i\phi_q), q = u, d, s$ such that the potential (2) takes the form:

$$V(\theta, \phi_i) = -E \cos \left(\frac{\theta - \sum \phi_i}{N_c} \right) - \sum M_i \cos \phi_i, \quad (4)$$

where M_i are the diagonal entries of the quark mass matrix which was introduced above. Notice that in Eq.(4) the θ parameter appears only in the combination $\sum \phi_i - \theta$. The minimum of the potential is given by the solution to the following equations:

$$\sin \left(\frac{\theta - \sum \phi_i}{N_c} \right) = \frac{N_c M_i}{E} \sin \phi_i. \quad (5)$$

In the limit where all the quark masses are equal and $M_i \ll E$, the approximate solution is given by $\phi_i \sim \frac{\theta}{N_f}$. In particular, in this limit and for $N_f = 2$, the vacuum energy $V_{vac}(\theta)$ can be approximated as follows:

$$\begin{aligned} V_{vac}(\theta) &\simeq V_{vac}(\theta = 0) - 2m|\langle \bar{q}q \rangle|(|\cos \frac{\theta}{2}| - 1), \\ \langle \theta | \frac{\alpha_s}{8\pi} G\tilde{G} | \theta \rangle &= -\frac{\partial V_{vac}(\theta)}{\partial \theta} = -m|\langle 0 | \bar{q}q | 0 \rangle| \sin \frac{\theta}{2}, \end{aligned} \quad (6)$$

which was used in the Introduction for estimates. From this we see that a θ^{ind} -state is degenerate with the $\theta = 0$ state in the chiral limit where $m_q = 0$, as expected.

3 Induced θ -Vacua

It was recently argued [1] that it may be possible to create an arbitrary $\theta \neq 0$ state in heavy ion collisions. In this scenario, we propose that the θ^{ind} -state is separated from our world ($\theta = 0$) by a shell of debris expanding out at the speed of light. This idea is similar to the so called disoriented chiral condensate that has been extensively studied in the last ten years [3]-[6].

We would like to stress the point that an induced θ -parameter is very different from θ^{fund} which is zero in our world and which cannot be changed. The simplest way to visualize θ^{ind} is to assume that right after the QCD phase transition the flavor singlet phase of the chiral condensate is non-zero in a macroscopically large domain. This phase is identified with θ^{ind} . This identification is a direct consequence of the transformation properties of the fundamental QCD Lagrangian under $U(1)_A$ rotations by which the chiral singlet phase can be rotated away at the cost of introducing θ^{ind} . From now on we will refer to a θ^{ind} -state as simply a θ -state, omitting the “ind” label.

Numerical lattice calculations in the past have provided strong evidence that QCD undergoes a phase transition at a temperature in the range of 150–200 MeV. It is believed that above this critical point that chiral symmetry is restored and/or deconfinement occurs (quark-gluon plasma). The idea of the Disoriented Chiral Condensate (DCC) arises when we consider what happens immediately after the phase transition upon cooling within heavy ion collisions. DCC refers to regions of space (interior) where the chiral condensate points in a different direction from that of the ground state (exterior), and separated from the latter by a hot shell of debris. The difference in energy between the created state and the lowest energy state is proportional to the small parameter m_q (see Eq.(6)) for both the DCC and the θ -state and is therefore negligible at high temperature.

First, we recall the DCC scenario as given in [5] with emphasis on the analogy between DCC and θ^{ind} -state. Rajagopal and Wilczek [5] use the $O(4)$ linear sigma model to describe the low energy dynamics of the pions and the chiral condensate. All fields are represented by a 4-vector with components $\phi = (\sigma, \vec{\pi})$, where σ represents the chiral condensate and $\vec{\pi}$ represents the triplet of pions. Throughout the exterior region the vacuum expectation value of ϕ is $(v, 0)$. In the interior region, however, the pion fields can become non-zero and $(\sigma, \vec{\pi})$ wanders in the four dimensional configuration space. The high energy products (shell of debris) of the collision expand outwards at relativistic speeds and separates the misaligned vacuum interior from the exterior region.

In [5] all calculations were done under the assumption of a quenched system. In the quenched approximation, the $(\sigma, \vec{\pi})$ fields are suddenly removed from a heat bath ($T \geq T_c \sim 200 \text{ MeV}$) and evolved according to the zero temperature equations of motion. This is done numerically by giving the fields a non-zero vacuum expectation value $\langle \phi \rangle \neq 0$ and letting this field configuration evolve in time according to the zero temperature equations of motion. The hope is that regions of misaligned vacuum with an arbitrary isospin direction will be created. In [5] it was shown that if the cooling process is very rapid and the system is initially out of equilibrium, there is a temporary growth of long wavelength spatial modes of the pion field corresponding to domains where the chiral condensate is approximately correlated. In the case of DCC, the created state will relax to the true vacuum by coherent emission of pions with the same isospin orientation producing clusters of charged or neutral pions. To be more specific, let us consider the case $N_f = 2$. The matrix U is parameterized by the misalignment angle ϕ and the unit vector \vec{n} in the isospin space:

$$U = e^{i\phi(\vec{n}\vec{\tau})}, UU^\dagger = 1, \langle \bar{\Psi}_L^i \Psi_R^j \rangle = -|\langle \bar{\Psi}_L \Psi_R \rangle| U_{ij}.$$

The energy density of the DCC is determined by the mass term:

$$E_\phi = -\frac{1}{2} \text{Tr}(MU + M^\dagger U^\dagger) = -2m|\langle \bar{\Psi}\Psi \rangle| \cos(\phi). \quad (7)$$

Eq. (7) implies that any $\phi \neq 0$ is not a stable vacuum state because $\frac{dE}{d\phi} \neq 0$, i.e. the vacuum is misaligned. Since the energy difference between the misaligned state and the true vacuum is proportional to m_q , the probability to create a state with an arbitrary ϕ at high temperature $T \sim T_c$ is proportional to $\exp[V(E_\theta - E_0)/T]$ and depends on ϕ only very weakly, i.e. ϕ is a quasi-flat direction. Right after the phase transition when $\langle \bar{\Psi}\Psi \rangle$ becomes non-zero, the pion field oscillates.

Now we turn to our main subject, the θ -vacuum state. First we would like to comment that even though the θ parameter is related to the large constant E in Eq. (4), θ dependence is actually proportional to m_q since in the chiral limit all dependence can be removed by performing a $U(1)_A$ rotation of the chiral condensate phases. When the combination of $(\sum \phi_i - \theta)$ is close by an amount $O(m_q)$ to its vacuum value, the Boltzmann suppression due to the term E at high temperature is absent. This is essentially what allows the induced θ -state to be formed.

To model the creation of an arbitrary θ -state, we will use the effective chiral Lagrangian described in the previous Sect. 2. We consider three flavors of massive quarks, and derive the following equation of motions:

$$\ddot{\phi}_i + \nabla^2 \phi_i + E \sin\left(\frac{-\theta + \sum \phi_i}{N_c}\right) - M_i \sin(\phi_i) - \gamma \dot{\phi}_i = 0, \quad i = u, d, s, \quad (8)$$

where the constant γ in front of $\dot{\phi}$ represents the damping of the system which can be attributed to various sources such as expansion or the emission of pions.

In order to model the situation numerically, several assumptions are made. First, assume that the θ^{ind} parameter acquires a non-zero value when the temperature $T \gg m_q$ and the system is out-of-equilibrium. Once again, $\theta^{ind} \neq 0$ follows from the assumption that the mean value of singlet phase $\langle \sum \phi_i \rangle$ is not zero over a macroscopically large domain

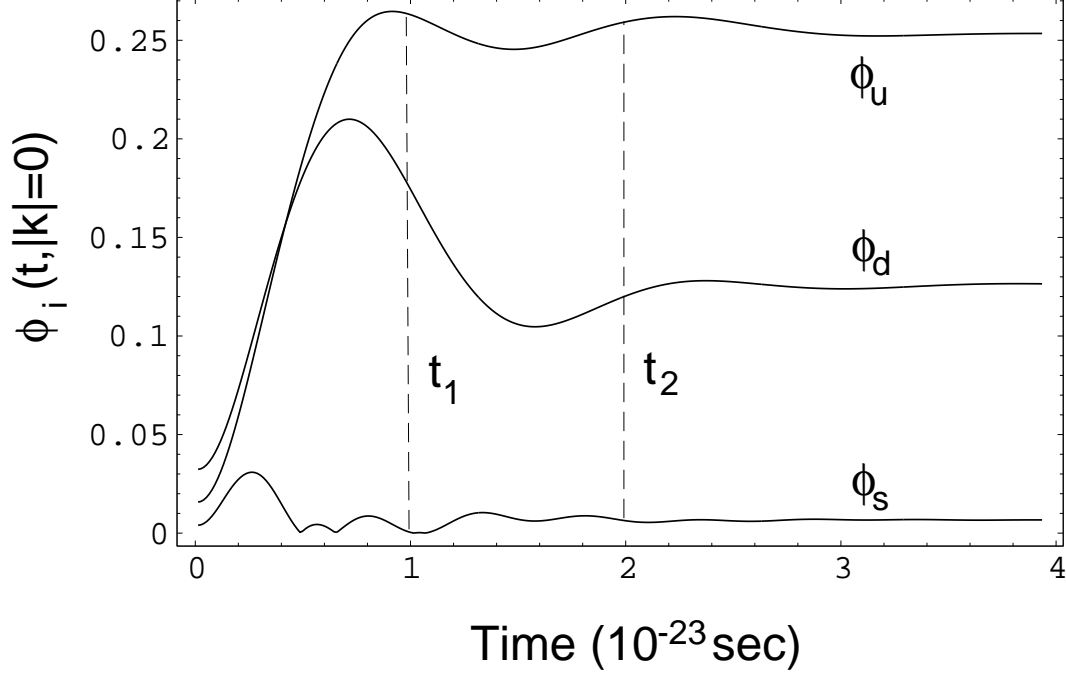


Figure 1: $|\phi_i(k=0)|$ is plotted as a function of time for the up, down, and strange quark. Notice that the zero momentum modes of the ϕ_i fields settle to a non-zero value in a time on the order of 10^{-23} s. The times t_1 and t_2 represent the values we chose for τ_{shell} , the time when the shell separating the two regions disappears.

immediately upon cooling through the phase transition. Performing a $U(1)_A$ rotation then gives us an equivalent description with $\theta^{ind} \neq 0$. The second assumption is that the phases ϕ_u , ϕ_d , and ϕ_s have small random values (modulo $\pi/3$) at these high temperatures. The final assumption is that the cooling process is very rapid and the system obeys the zero temperature equations of motion after this initial quench. There is obviously some sort of damping mechanism present in any real system, and this damping can be attributed to the expansion of the region inside the shell and/or the radiation of Goldstone bosons. The damping constant, γ , was chosen to be of the same order of magnitude as Λ_{QCD} , which should be a reasonable value according to all other scales in the system.

The numerical algorithm used to solve these coupled differential equations was a fourth-order Adams-Bashforth-Moulton predictor-corrector method. The phases were initially assigned random values according to a uniform distribution ($|\phi_i| < \pi/3$). The phases were rescaled in such a way as to be dimensionless. We evolved the equations for 6000 time steps of size 10^{-27} sec on a cubic lattice with spacing $a = 0.005 \text{ MeV}^{-1} = 1 \text{ fm}$. In order to examine the momentum dependence, the Fourier transform [15] of the field configuration was calculated at periodic intervals in time. The data was then binned according to the magnitude of the wave vector \vec{k} so that the dependence on $|\vec{k}|$ could be examined. We consider a volume of $(8 \text{ fm})^3$. In all calculations that follow, we use the current quark masses $m_u = 4 \text{ MeV}$, $m_d = 8 \text{ MeV}$, and $m_s = 150 \text{ MeV}$.

In Fig. 1 we show that the zero modes of the ϕ_i fields settle down to a constant value in a time $\tau \sim 10^{-23}$ s. In the same time interval, all other modes die off. This, along with

a test of volume independence, shows that a true non-perturbative condensate has been formed. In the case where all quark masses are equal, the zero mode is expected to settle down to $\phi_i \sim \theta/N_f$, which has also been verified numerically.

In order to incorporate the effects of expansion, we could introduce a time dependent lattice spacing $a(t)$. If $a(t)$ increases with time, this will model the dilution of energy in the system. Following an analogy with an expanding universe, the coefficient of the damping term $\dot{\phi}_i$, γ goes like $constant \times \dot{a}/a$. Although we have not taken this into account in our analysis, we can estimate the result as follows. The Laplacian term in Eq.(8) is numerically proportional to $1/a$, and if $a(t)$ increases with time, the system will require more time to relax to the θ -state. If the coefficient, γ , is altered as mentioned above and $a(t)$ increases at a constant rate, the damping term will become smaller as time goes on. The overall effect of a time dependent lattice spacing is to increase the formation time of the θ -state.

4 Goldstone Bosons

Now that we have argued the induced θ -state may be formed in heavy ion collisions we would like to discuss how to detect them experimentally. Naturally, the first thing to look for would be axion production. If axions[14] do exist, they can be produced during the relaxation of $\theta^{ind} \neq 0$ to the lowest energy state with $\theta = 0$. However, the axion production rate under the conditions which can be achieved at RHIC will be too low in comparison with the limit already achieved from the astrophysical and cosmological considerations[14]. Therefore, the question remains: “How could the the formation of a θ -state be detected experimentally?” Fortunately there are a number of ways in which this might be done. One proposed possible signature is related to the observation of specific CP odd correlations of the Goldstone bosons [7] but the effect would probably be washed out by final state interactions. A second possible signature was proposed in [8] which comes about because of the unusual properties of the Goldstone bosons and η' in the θ -background. In the θ -world the masses of these particles are altered and for example the mass of π^0 would be noticeably smaller. As well, these particles become a mixture of pseudoscalar/scalar rather than pure pseudoscalars as for $\theta = 0$. This fact means that the CP odd decays $\eta \rightarrow \pi\pi$ and $\eta' \rightarrow \pi\pi$ are no longer suppressed in the θ background and become of order 1 [8]. The full width of the η', η decays in our world are much smaller than the above decays in the θ -world:

$$\Gamma_{\theta \neq 0}(\eta' \rightarrow \pi\pi) \sim 2 \text{ MeV} \gg \Gamma_{\theta=0}^{total}(\eta') \sim 0.2 \text{ MeV (experimental)}, \quad (9)$$

$$\Gamma_{\theta \neq 0}(\eta \rightarrow \pi\pi) \sim 0.2 \text{ MeV} \gg \Gamma_{\theta=0}^{total}(\eta) \sim 118 \text{ keV (experimental)}. \quad (10)$$

Thus observation of an increased rate of decay of η', η at masses slightly shifted from the accepted values would be a signature of the θ -vacuum. However, this signal may be a difficult one to distinguish in the large amounts of data generated in heavy ion collisions. Therefore we would like to discuss a signature that would be easier to experimentally observe. In this work we consider an additional new signature which could possibly verify if a θ -state is created in heavy ion collisions. As was discussed briefly in the introduction,

the creation of a θ -state could greatly enhance the production of low momentum ($\sim 10 \text{ MeV} - 100 \text{ MeV}$) Goldstone bosons.

Before going into the details we would like to discuss the concepts behind this idea. We have already presented some arguments that a θ -state may be formed in heavy ion collisions if protected from the exterior $\theta = 0$ world by the shell of debris. At some point, however, this shell ceases to isolate the interior and the influence of the exterior world will be felt. The way in which this happens is not well understood, but we would like to use an instantaneous approximation in order to obtain some feeling about the possible outcome. There are basically two time scales present in the system: the lifetime τ_{shell} of the shell separating the two worlds and the scale associated with strongly interacting particles, $(\Lambda_{QCD})^{-1}$. If these differ by orders of magnitude, the disappearance of the shell can be considered either as an instantaneous perturbation or an adiabatic perturbation.

Realistically, we expect τ_{shell} to be about the same order of magnitude as $(\Lambda_{QCD})^{-1}$ and therefore the disappearance of the shell is somewhere between an instantaneous process and an adiabatic one. However, we do not yet know how to treat the process properly so we will use the instantaneous approximation in order to obtain an order of magnitude estimate of the spectrum of emitted particles.

The instantaneous perturbation is an approximation that is well known in quantum mechanics. For our specific problem we essentially assume that all states which were formed in the θ^{ind} background will suddenly find themselves in the new vacuum state (with $\theta = 0$) after the shell *instantaneously* blows apart. These states have no choice but to transform to the asymptotic states of this new (for them) $\theta = 0$ world. The dynamics of this strong transformation is quite complicated and beyond the scope of the present work. However, if this transition is instantaneous, the procedure to do the calculations for such a transition is well known: we assume that the initial state has no time to adjust its properties when the shell decays, and therefore, we expand the initial state in terms of the asymptotic states of $\theta = 0$ world.

In more detail, we do the following. The field values, ϕ_i , obtained for the θ -state as in previous work [1] are embedded into a larger grid where the field values take their $\theta = 0$ vacuum values. This data now contains a plateau of field values corresponding to θ -state values surrounded by zero field values. This field configuration must now resolve itself into asymptotic free particles. With this process in mind we determine the momentum spectrum of free particles corresponding to this distribution by considering the Goldstone boson fields as true quantum fields at this instant.

In order to calculate the spectrum, we must obtain the quantities $a(\vec{k})$ and $a^\dagger(\vec{k})$. These are obtained by performing the Fourier transform of the field configuration at time $t = \tau_{shell}$ when the shell breaks down,

$$a(\vec{k}) = \int d^3\vec{x} \psi(\vec{x}) \exp(-i\vec{k} \cdot \vec{x}), \quad (11)$$

where $\psi(\vec{x})$ is the distribution amplitude of the ϕ_i fields obtained from evolving the equations of motion (Eq.(8)) on a cubic lattice, as was done in [1]. From Eq.(11), the number operator is given by $N(\vec{k}) = N_0 a(\vec{k}) a^\dagger(\vec{k})$ where N_0 is an overall constant that will be determined using the following argument. As we mentioned earlier, the θ -state differs from the true vacuum state by a small amount of energy $\sim m_q$. The amount of energy

that is available when the θ -vacua decays is obtained by analyzing the θ dependence of the vacuum energy density E :

$$E_\theta = m_q |\langle \bar{\Psi} \Psi \rangle| N_f \cos\left(\frac{\theta}{N_f}\right). \quad (12)$$

Therefore, the amount of energy $\Delta E V$ available due to the formation of the θ -state as discussed above is given by

$$\Delta E V = (E_{\theta=0} - E_\theta) \cdot V \simeq 20 \left(\frac{V}{(10 fm)^3} \right) GeV, \quad (13)$$

where V is the the volume of the created θ -region. If we take for example a θ -state with volume $V = (8 fm)^3$ the amount of energy available is $\Delta E V = 10 GeV$. This represents only a small amount of energy compared to the cm energy, $\sqrt{s} = 40 TeV$, expected in heavy ion collisions. If the total amount of available energy is known, the constant N_0 can be fixed by enforcing the following conservation of energy constraint

$$\Delta E V = \sum_i \int \frac{d^3 \vec{k}}{2(2\pi)^3} N_0 a_i(\vec{k}) a_i^\dagger(\vec{k}), \quad (14)$$

where the sum is performed over all types of particles considered (*ie.* all neutral Goldstone bosons). Once this constant N_0 is fixed, the total number of pions and etas can be calculated by

$$N = \int \frac{d^3 \vec{k}}{2\omega_k (2\pi)^3} N_0 a(\vec{k}) a^\dagger(\vec{k}), \quad (15)$$

for the simplified case of only one collision of two nuclei. If we consider that RHIC will collide gold nuclei at a rate of central collisions of about 1 kHz , we can estimate the total number of these low momentum particles produced by simple multiplication. In what follows we present all results for just one event.

As we have already described, we proceed as follows. At some time t before the fields ϕ_i settle down to the constant field configuration (see Fig. 1), we take the position space data and embed this in a larger square grid where the field values are zero (our world). We assume an instantaneous perturbation and then take the fourier transform of this field configuration in order to obtain the operators $a(\vec{k})$ and $a^\dagger(\vec{k})$. It may be argued that the instantaneous approximation is not completely justified in this case as we expect $\tau_{shell} \sim \tau_{QCD}$ and therefore this calculation gives us a rough estimate at best.

In order to analyze the spectrum of the diagonal (neutral) components of the matrix $U(\pi^0, \eta, \eta')$ we must analyze the following combination of the ϕ'_i s:

$$\begin{aligned} \pi^0 &= \frac{f_\pi}{2\sqrt{2}}(\phi_u - \phi_d), \\ \eta &= \frac{f_\pi}{2\sqrt{6}}(\phi_u + \phi_d - 2\phi_s), \\ \eta' &= \frac{f_{\eta'}}{2\sqrt{3}}(\phi_u + \phi_d + \phi_s). \end{aligned} \quad (16)$$

For all numerical calculations, we can take the constant $f_{\eta'}$ to be $\sim f_{\pi}$. One of the crucial decisions which must be made is when to choose the time at which the shell breaks down. We chose several values for τ_{shell} . The majority of the particles which are produced due to the formation of the $|\theta\rangle$ -state have momentum $k < 25 MeV$ (for our choice of the volume $V = (8 fm)^3$). In Fig. 2 we show a plot of $N_{\pi^0}(k)$ as a function of $|\vec{k}|$ for the neutral pion. The function $N_{\pi^0}(k)$ has dimensions MeV^{-1} , so that the total number of particles, N_{π^0} , is a dimensionless number given by:

$$\begin{aligned} N_{\pi^0} &= \int_{-\infty}^{\infty} dk N_{\pi^0}(k) \\ &= \int_{-\infty}^{\infty} dk \frac{4\pi}{2(2\pi)^3 w_k} N_0 a(\vec{k}) a^\dagger(\vec{k}). \end{aligned} \quad (17)$$

The two different lines represent the times at which we assumed the shell to break down. The solid line represents the earlier time ($\tau_{shell} = t_1 = 1500/6000$ time steps) while the dotted line represents $\tau_{shell} = t_2 = 3000/6000$ time steps (this representation for t_1 and t_2 will be followed in all graphs that follow). For this calculation, we assumed the created θ -state had a volume of $(8 fm)^3$ which exists in a larger volume of $(64 fm)^3$. The explanation of this phenomenon is simple: at time t_1 the amplification of the zero mode is not as profound as at time t_2 . Furthermore, at time t_1 a considerable portion of the energy goes to high momentum particles.

Below we show the same graph for the η and the η' . Notice that the η' is produced in large amounts compared to the π^0 -mesons and the η -mesons. In order to obtain the total number of each particle produced, we use Eq.(15) to calculate the total number of particles produced per collision: $N_{\pi^0} = 0.1$, $N_{\eta} = 2.8$, $N_{\eta'} = 19.3$ for $t = t_1$ and $N_{\pi^0} = 0.5$, $N_{\eta} = 4.3$, $N_{\eta'} = 18.4$ for $t = t_2$. We have also checked that in the limit where all quark masses are equal, only the η' is produced, as expected. It should be noted that, as demonstrated by these figures and the coarsening phenomenon (i.e. the phenomenon of amplification of the zero mode as time increases), we would expect that as τ_{shell} increases up to a maximum value, the majority of the particles would reside in the low momentum regime. We consider our estimate as a very conservative one due to the fact that we do not count the particles η and η' which will be produced during the formation period of the θ^{ind} state. These particles will have higher momentum modes and can not be easily distinguished from the large number of the similar particles from the background. Therefore, the main conclusion of our analysis is as follows: if induced θ state is produced than it will result in the strong enhancement of emission of low-energy η and η' mesons which clearly signals the existence of the induced θ state.

When our analysis was completed, we became aware that a similar conclusion has been reached in a recent paper by Baier *et al.* [16] where the production of an excess of η' -mesons is discussed in the scenario suggested in [7]. Despite the fact that the starting points of the present work and [16] are quite different (we start from the induced θ state while in paper [16] the starting point is a metastable state which acts like a region with a non-vanishing θ angle), the general conclusion is very similar. Namely, a strong enhancement of emission of low-energy η' -mesons is expected irrespectively of the origin of the initial unusual state. The details, however, are quite different. The main difference is the following. We assume that a θ -state is produced and instantaneously decays such

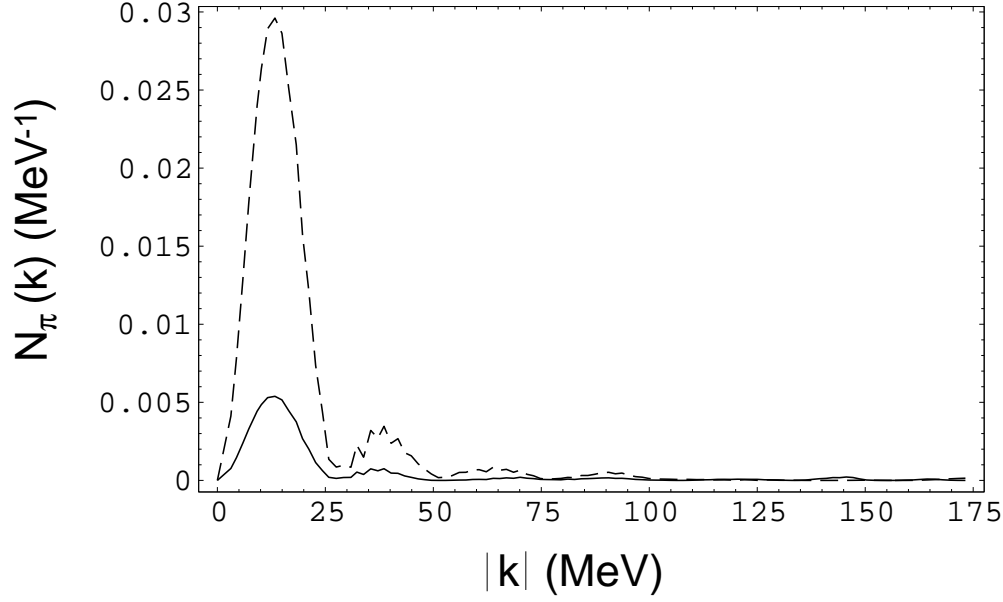


Figure 2: The number of neutral pions produced, $N_{\pi^0}(k)$, is plotted as a function of the magnitude of the wave vector, $|\vec{k}|$. Above we show that the momentum distribution of the π^0 -mesons produced is primarily $< 25 \text{ MeV}$ for two different values of τ_{shell} . The solid line represents the earlier time t_1 and the dotted line represents the later time t_2 (see Fig. 1 for the positions of t_1 and t_2 in the evolution of the ϕ_i fields). This is expected as the formation of the θ -state can be attributed to the enhancement of the low momentum modes.

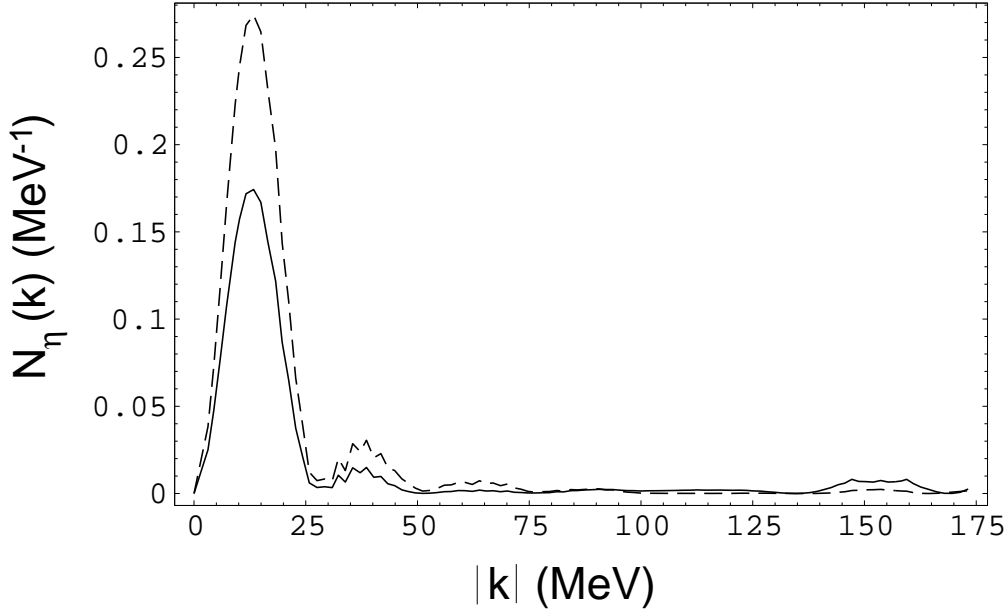


Figure 3: $N_{\eta}(K)$ is plotted as a function $|\vec{k}|$, at two different times t_1 and t_2 as in Fig. 2. The higher peak represents the later time with the main difference being that now a larger percentage of the produced particles lie in the $k < 25 \text{ MeV}$ range.

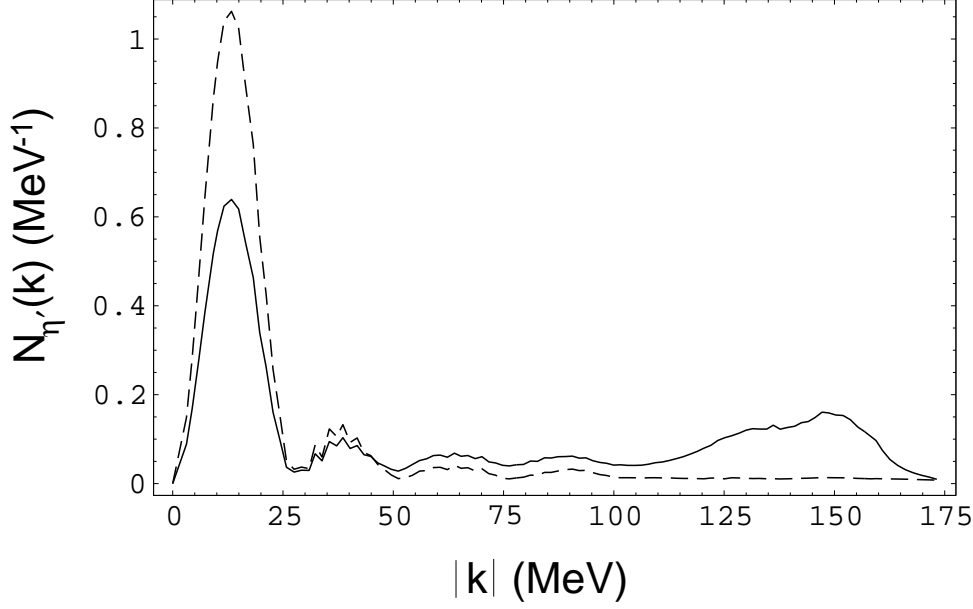


Figure 4: $N_{\eta'}(k)$ vs. $|\vec{k}|$ is shown above. This graph shows that the η' clearly dominates the spectrum compared to the other neutral particles.

that we calculate the particle production due to this single event, not taking into account previous particle production during the formation period. As was mentioned above, we expect that those particles will have much higher momentum modes and we ignore them. The approach of [16] is different from ours as their calculation of the number of produced η' -mesons is based on the specific mechanism of the decay of the original metastable state [7]. Such a decay receives contributions from the whole evolution, not just one particular instant. This difference explains the difference in rates: for a domain with radius 5 fm , they estimate that about 90-100 η' -mesons would be produced while our estimation is 4 – 5 times smaller.

In the above calculations of $N(\vec{k})$ the geometry of the created θ -state is assumed to be a cubical box. For the case of heavy ion collisions, the more realistic case is that of an ellipsoid or elongated rectangular box. Upon collision, we expect the collision to create a region of quark-gluon plasma which has an asymmetry in one direction. In order to model this situation, we evolve a rectangular grid and embed it in a larger square grid and compute the Fourier transform. Once again, we consider the angular averaged value of $N(\vec{k})$.

For a θ -region with dimensions $32 \text{ fm} \times (8 \text{ fm})^2$, we show the spectrum of the η' at the same times as shown in Fig. 4. The side of length 32 fm is the direction parallel to the beam direction. The low momentum modes still dominate, but the peak is not as sharp and the higher modes show a stronger presence. Also, since the θ -region is now larger, from Eq.(13) we would expect more particles to be created since there is now more energy available. This is evident when comparing Fig. 4 with Fig. 5. For the two different times shown, we find that $N_{\pi^0} = 0.3$, $N_{\eta} = 12.7$, $N_{\eta'} = 76.3$ for $t = t_1$ and $N_{\pi^0} = 1.7$, $N_{\eta} = 16.8$, $N_{\eta'} = 73.8$ for $t = t_2$.

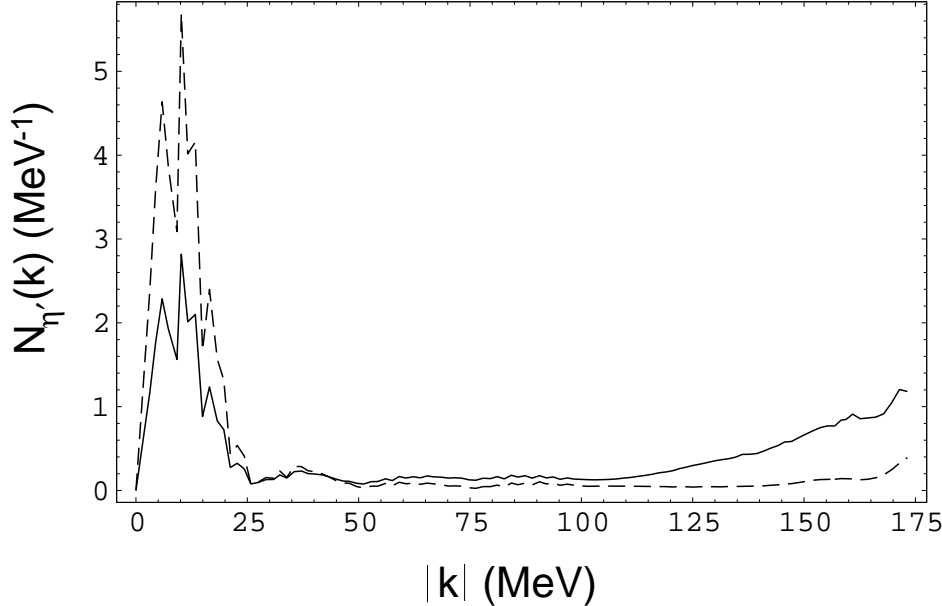


Figure 5: The momentum distribution ($N_{\eta'}(k)$ vs. $|\vec{k}|$) is now shown for the case where the geometry of the θ -region is considered as a rectangle. We consider the case where the size of the θ -region is larger along the longitudinal directions compared to the transverse directions. The solid and dotted lines once again represent times $\tau_{shell} = t_1$ and t_2 respectively. Although the majority of the spectrum still exists in the low momentum range, the higher momentum modes now become more evident.

In this case where spherical symmetry is lost, we must also examine the angular distribution of the system. We choose the coordinates such that the long edge of the θ -region is parallel to the z -axis. In Fig. 6, we show the angular distribution of $N_{\pi^0}(\theta)$ as a function of the azimuthal angle θ . This plot suggests that the majority of the particles would have larger momentum components in the direction perpendicular to the beam direction.

As was shown in Figs. 2,3,4, and 5, if a θ -state can be created in heavy ion collisions we should expect an excess of low momentum Goldstone bosons. In particular, the η' would be produced in large amounts. Besides that, the low momentum π^0 , η , and η' -mesons would considerably increase the photon and e^+e^- pair production through such decays as $\pi^0 \rightarrow \gamma e^+e^-$, $\eta \rightarrow \gamma e^+e^-$, and $\eta' \rightarrow \gamma e^+e^-$. In particular, it would result in enhancement of low-energy dileptons, which could possibly provide a solution to the observation of an excess of these particles seen at CERN [9]. Indeed, as was demonstrated by Kapusta *et al.* in [17] a sufficiently large enhancement² of η' -production could easily explain the excess of dileptons seen at CERN [9]. The large amount of produced η' -mesons in our work is a direct consequence of the decay of the induced θ -state.

²The enhancement mechanism of η' -production suggested in [17] is quite different from what we have considered in this paper. However, the general conclusion (that η' enhancement leads to the excess of dileptons) is insensitive to the nature of the η' -mesons. Only by performing a detailed experimental analysis of the momentum and angular distributions can one mechanism be chosen over another.

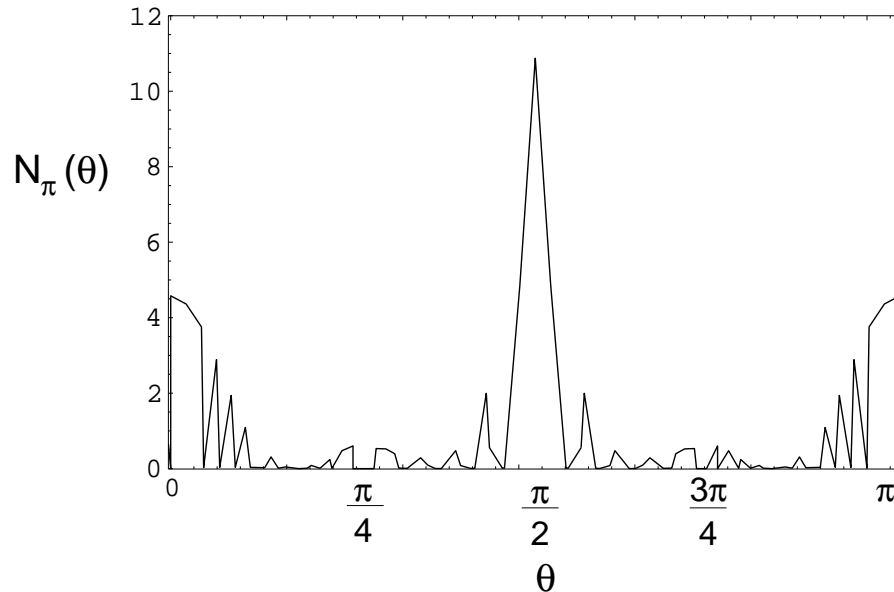


Figure 6: For the scenario shown in Fig. 5, we consider the angular dependence on the azimuthal angle for the π^0 instead of the η' . The azimuthal angle is defined so that $\theta = 0$ coincides with the beam axis.

5 Conclusion

In this work we showed that the consequences of creating non-trivial θ -vacua in heavy ion collisions could be the amplification of production of light Goldstone bosons in the 10 *MeV* momentum range. In all calculations, we worked in the instantaneous approximation where the shell separating the θ -state disappears in a time much less than any internal time scale. These low momentum particles could possibly then decay to low momentum photons and dileptons, which could be easily detected. We would like to make the suggestion that this could possibly account for the unexplained abundance of low momentum dileptons observed at CERN [9]. We are thankful to Dima Kharzeev for correspondence regarding the relevant work in [17].

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